Salt Lake Community College
Math 1060 Final
Spring 2014

Part I – No Calculator

1. Find the exact value of each of the following or state that it is undefined.

   a) $\sin(-30°) = -1/2$

   b) $\cos(45°) = \frac{\sqrt{2}}{2}$

   c) $\cot(90°) =$ undefined

2. Find the exact value of each of the following or state that it is undefined.

   a) $\tan\left(\frac{15\pi}{4}\right) = -1$

   b) $\sec\left(-\frac{2\pi}{3}\right) = -2$

   c) $\csc\left(-\frac{\pi}{4}\right) = -\sqrt{2}$

3. Consider the function $y = 2\sin\left(\frac{\pi x}{4} + \pi\right) + 1$. $y = 2\sin\left[\frac{\pi}{4}(x+4)\right] + 1$

a) Find the phase shift.

left 4

b) Find the period.

$P = \frac{2\pi}{\pi/4} = 8$

c) Find the frequency. $f = \frac{1}{8}$

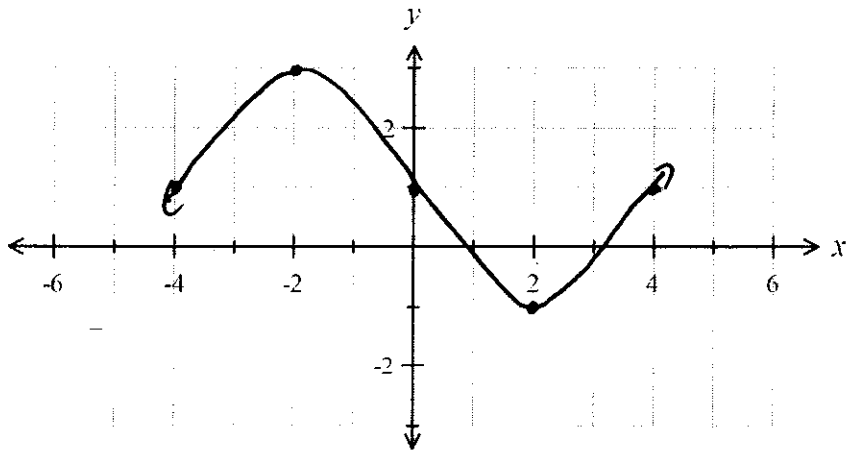
d) Find the amplitude. $a = 2$

e) Find the range. Range: $[-1, 3]$

f) Graph at least one period of the function and give the coordinates of the five key points on the graph.

parent table

x	y
0	0
$\pi/2$	1
π	0
$\frac{3\pi}{2}$	-1
2π	0



$\frac{4}{\pi}x - 4$	$2y+1$
-4	1
-2	3
0	1
2	-1
4	1

← key points

4. a) Complete the formula: $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \sin\beta\cos\alpha$

b) Use the formula from a) to find the exact value of $\sin\left(\frac{7\pi}{12}\right)$.

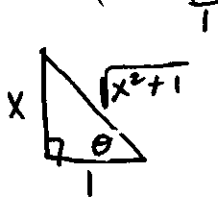
$$\frac{7\pi}{12} = \frac{3\pi}{12} + \frac{4\pi}{12}$$
$$= \frac{\pi}{4} + \frac{\pi}{3}$$
$$\sin\frac{\pi}{4}\cdot\cos\frac{\pi}{3} + \sin\frac{\pi}{3}\cdot\cos\frac{\pi}{4}$$
$$\frac{\sqrt{2}}{2}\cdot\frac{1}{2} + \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} = \boxed{\frac{\sqrt{2}+\sqrt{6}}{4}}$$

5. Find all exact real number solutions in the interval $[0, 2\pi)$ of the equation $2\cos(2x)+1=0$.

$$2\cos(2x)+1=0$$
$$\cos 2x = -1/2$$
$$\frac{2x}{2} = \frac{2\pi}{3} + 2\pi k \qquad \frac{2x}{2} = \frac{4\pi}{3} + 2\pi k$$

$$\boxed{x = \frac{\pi}{3}, \frac{4\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{3}}$$

6. Find an equivalent algebraic expression for $\sec(\arctan(x))$.



$$\sec\theta = \boxed{\sqrt{x^2+1}}$$

7. Evaluate each expression.

a) $\arccos\left(-\frac{\sqrt{3}}{2}\right)$ $= 150°$ or $\frac{5\pi}{6}$

b) $\tan\left(\arcsin\left(-\frac{\sqrt{2}}{2}\right)\right)$ $= -1$

$-\frac{\pi}{4}$

8. Prove that the equation is an identity.

$\tan^3 x = \tan x \sec^2 x - \tan x$

$= \tan x(\sec^2 x - 1)$

$= \tan x \cdot \tan^2 x$

$= \tan^3 x$ ✓

Part II – Calculator

9. Given the vectors $\mathbf{u} = \langle -2, 3 \rangle$ and $\mathbf{v} = \langle 1, -4 \rangle$,

a) Find $2\mathbf{u} - \mathbf{v}$.

$2\langle -2, 3 \rangle - \langle 1, -4 \rangle$

$\langle -4, 6 \rangle + \langle -1, 4 \rangle = \langle -5, 10 \rangle$

b) Find $|\mathbf{u}|$.

$|u| = \sqrt{(-2)^2 + (3)^2}$

$= \sqrt{4 + 9} = \sqrt{13}$

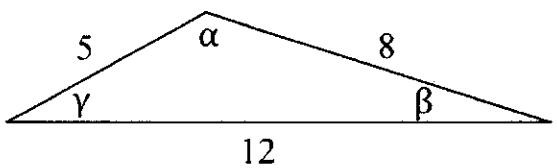
c) Are the vectors **u** and **v** orthogonal (perpendicular)? Provide a numerical justification.

$u \cdot v = (-2)(1) + (3)(-4)$

$= -2 + -12 = -14 \rightarrow$ not perpendicular

dot product not zero!

10. Solve the triangle. Round answers to the nearest hundredth of a degree.



$12^2 = 8^2 + 5^2 - 2(8)(5)\cos\alpha$

$144 = 64 + 25 - 80\cos\alpha$

$144 = 89 - 80\cos\alpha$

$-89 \quad -89$

$55 = -80\cos\alpha$

$\alpha = \cos^{-1}\left(\frac{55}{-80}\right) \approx 133.43°$

$\frac{\sin 133.43°}{12} = \frac{\sin\beta}{5}$

$\beta = \sin^{-1}\left(\frac{5\sin 133.43°}{12}\right) \approx 17.61°$

$\gamma = 180° - 133.43° - 17.61°$

$\gamma = 28.96°$

11. Suppose the tip of a lawnmower blade of diameter 20 inches long is spinning at 2,200 rev/min. Find the linear velocity in miles per hour. Round your answer to the nearest tenth. (1 mile = 5,280 feet)

$r = 10 in$

$$\frac{2{,}200\ rev}{1\ min} \cdot \frac{2\pi\ rad}{1\ rev} \cdot \frac{1\ ft}{12\ in} \cdot \frac{1\ mile}{5280\ ft} \cdot \frac{60\ min}{1\ hr} \cdot \frac{10\ in}{1\ rad}$$

130.9 mph

12. a) Write the complex number $(1-i)$ in trigonometric form. Express θ in degrees.

$r = \sqrt{1^2 + 1^2} = \sqrt{2}$ $\quad \theta = 315°$

$\sqrt{2}(\cos 315° + i \sin 315°)$

b) Use De Moivre's theorem to find $(1-i)^3$. Leave your answer in trigonometric form

$r = \sqrt{2}$ $\quad \theta = 315°$

$3 \cdot 315° = 225°$

$(\sqrt{2})^3 (\cos(3 \cdot 315°) + i \sin(3 \cdot 315°))$

$= 2\sqrt{2}(\cos 225° + i \sin 225°)$